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Consideration of statistical uncertainties for the determination of representative values of the specific activity of wastes

The German Radiation Protection Commission has recommended "Principles and Methods for the Consideration of Statistical Uncertainties for the Determination of Representative Values of the Specific Activity of NORM wastes" concerning the proof of compliance with supervision limits or dose standards according to §97 and §98 of the Radiation Protection Ordinance, respectively. The recommendation comprises a method ensuring the representativeness of estimates for the specific activity of NORM wastes, which also assures the required evidence for conformity with respect to supervision limits or dose standards, respectively. On the basis of a sampling survey, confidence limits for expectation values of specific activities are determined, which will be used to show that the supervision limit or the dose standard is met or exceeded with certainty, or that the performed sampling is not sufficient for the intended assessment. The sampling effort depends on the type and the width of the distribution of specific activities and is determined by the position of the confidence interval with respect to the supervision limit or of the resulting doses with respect to the dose standard. The statistical uncertainties that are described by confidence limits may be reduced by an optimised extension of the sample number, as far as necessary.

Berücksichtigung von statistischen Unsicherheiten für die Ermittlung repräsentativer Werte der spezifischen Aktivität von Rückständen. *Die deutsche Strahlenschutzkommission hat „Grundsätze und Methoden zur Berücksichtigung von statistischen Unsicherheiten für die Ermittlung repräsentativer Werte der spezifischen Aktivität von Rückständen" für den Nachweis der Einhaltung von Überwachungsgrenzen bzw. des Dosisrichtwertes gemäß StrlSchV §97 bzw. §98 empfohlen. Die Empfehlung enthält eine Methodik zur Gewährleistung der Repräsentativität von Werten der spezifischen Aktivität von Rückständen, die zugleich die geforderte Sicherheit des Nachweises der Einhaltung von Überwachungsgrenzen bzw. des Dosisrichtwertes gewährleistet. Anhand von Stichproben werden Konfidenzgrenzen für Erwartungswerte der spezifischen Aktivitäten bestimmt. Damit kann entweder eine sichere Unter- oder Überschreitung der Überwachungsgrenze bzw. des Dosisrichtwertes belegt werden oder es ist festzustellen, dass die realisierte Stichprobe für die beabsichtigte Bewertung noch nicht ausreicht. Der Aufwand zur Beprobung hängt von Art und Breite der statistischen Verteilung der spezifischen Aktivitäten ab und wird über die Lage des Konfidenzintervalls zur Überwachungsgrenze bzw. der resultierenden Dosis zum Dosisrichtwert bestimmt. Soweit nötig, können die durch Konfidenzgrenzen erfassten statistischen Unsicherheiten durch eine optimale Vergrößerung der Stichprobenzahl reduziert werden.*

1 Introduction

NORM wastes of type and origin specified in appendix XII part A of the German Radiation Protection Ordinance [1] are outside of regulatory control, if they comply with the supervision limits specified in appendix XII part B for certain ways of disposal and reuse. The compliance with a supervision limit C that is valid for a specified way of disposal or reuse has to be proven according to the condition

$$C_{U238max} + C_{Th232max} \leq C \qquad (1)$$

$C_{U238max}$ and $C_{Th232max}$ represent the maximum values of specific activities of the long-lived radionuclides of the U-238 and Th-232 decay chains, respectively, and have to be determined representatively. In Eq. (1), values of $C_{U238max}$ or $C_{Th232max}$ below 0.2 Bq/g are allowed to be neglected.

With respect to the verification of conformity with supervision limits or with the effective dose standard of 1 mSv/a according to §§97 et sqq. of the Radiation Protection Ordinance, the German Radiation Protection Commission has recommended principles and methods for the consideration of statistical uncertainties for the determination of representative values of the specific activity of wastes [2]. This recommendation comprises principles and methods for the specification of waste batches, of appropriate types of sampling, of the radionuclides for which the specific activity has to be measured or determined by means of activity ratios (according to former investigations), and also a new method for the confirmation of representativeness of values of the specific activity. These principles and methods ensure evidence of the compliance with the surveillance limit or the dose standard of 1 mSv/a for individuals of the public.

Even in waste batches that are well specified with respect to their origin and amount, the specific activity may vary considerably between particular parts/samples. In addition, the batch volume is allowed to be specified according to actual circumstances, by which the degree of heterogeneity or the temporal variation of the specific activity in a batch can be influenced.

From the methodical point of view, the broad spectrum of waste types defined in appendix XII part A of the Radiation Protection Ordinance had to be taken into account. From case to case, the specific activity of the waste can be relatively homogeneous or feature pronounced spatial or temporal variations about orders of magnitude. The average specific activity of the long-lived radionuclides relevant for the radiological assessment of the waste can be in the vicinity of or far below/above the supervision limit. Respectively, the corresponding dose estimated according to §98 of the Radiation Protection Ordinance may be near or far below/above the dose standard.

For the determination of representative values of the specific activity, these cases have to be distinguished in order to avoid unnecessary expenses for measurements, and, on the other hand, to ensure the required certainty of the radiological assessment.

In cases where supervision limits or the dose standard defined in the Radiation Protection Ordinance are considerably exceeded or under-run, greater uncertainties of the estimated expectation value of the specific activity are acceptable, as far as they are not relevant for the decision on waste management. In contrast, if the sum of the specific activities $C_{U238max}$ and $C_{Th232max}$ is near to the supervision limit C, it may be necessary to reduce the statistical uncertainties by increasing the number of samples in order to enable a reliable decision. Therefore, the representativeness of sampling depends both on the waste characteristics and the inspection value to be applied.

2 Inadequacy of point estimators

Point estimators of the expectation value like the arithmetic mean or the maximum-likelihood estimate, calculated for a certain sample number, are random variables. The resulting statistical uncertainty of the estimated expectation value depends both on the actual type and width of the distribution of the specific activity and of the sample number applied. Thus, based on point estimators for the expectation value (e.g., the arithmetic mean or the maximum-likelihood average value), it is impossible to appraise neither the representativeness of the underlying sample nor the reliability of derived conclusions.

To illustrate the uncertainty of point estimators for the expectation value, two lognormal distributed variables $X = \mathrm{Ln}(\mu, \sigma)$ with the parameter values

$\mu = -1{,}4817;\ \sigma = 1{,}5$ and

$\mu = -0{,}8626;\ \sigma = 1{,}5$

are considered. The (true) expectation values are E = 0.7 Bq/g and E = 1.3 Bq/g, respectively;

$$E = \exp\,(\mu + \sigma^2/2) \qquad (2)$$

Fig. 1 shows, for these examples, the cumulative distributions of maximum-likelihood estimates of the expectation value based on a sampling number of n = 10.

Supposed that a supervision limit of C = 1 Bq/g has to be applied, in the case with the (true) expectation value of E = 0.7 Bq/g, the probability of an erroneous implication that the supervision limit would be exceeded is 29 %. In the case with the (true) expectation value of E = 1.3 Bq/g, the probability of an erroneous implication of compliance with the supervision limit is 37 %. Using the arithmetic mean for estimating the expectation value, the corresponding error probabilities are 17 % and 50 %, respectively.

3 Concept of interval estimators

For the proof of compliance with supervision limits of the specific activity or the dose standard, the deficiency of point estimators illustrated above can be avoided by the application of interval estimators. In the framework of this concept, based on the specific activity values measured for a certain number of samples, a confidence interval is calculated, which covers with a preassigned probability the true (never exactly known) expectation value of the specific activity of the batch. For all long-lived radionuclides (r), one calculates upper confidence limits UL_r, which are greater than the true expectation values E_r with confidence level P. Similarly, lower confidence limits LL_r have to be calculated, which are less than the true expectation values E_r with confidence level P. To forecast the increase of the sample number, which may be necessary to prove the compliance/disagreement with a supervision limit or with the dose standard, the calculation of most probable expectation values PL_r of the specific activity is advisable.

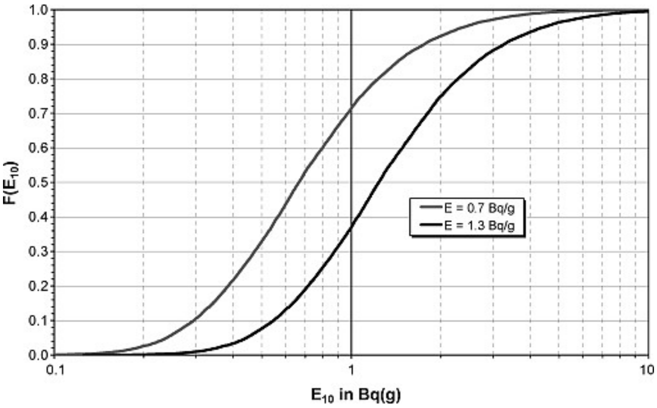

Fig. 1. Cumulative distributions of maximum-likelihood estimates of the expectation value of lognormal distributed variables with σ = 1.5 using a sampling number of n = 10; grey: case with the true expectation value of E = 0.7 Bq/g; black: case with the true expectation value of E = 1.3 Bq/g

In [2], for a reliable prove of compliance/disagreement with defined supervision limits or dose standards, the application of a confidence level of P = 0.95 has been recommended by the German Radiation Protection Commission.

For the prove of compliance with a supervision limit specified in appendix XII part B of the German Radiation Protection Ordinance, the "representative" maximum values $C_{U238max}$ and $C_{Th232max}$ of specific activities of the long-lived radionuclides of the U-238 and Th-232 decay chains have to be defined on the basis of the above defined upper confidence limits:

$$C_{U238\,max} = \max_{r \in U238-chain}\{UL_r\}, \quad C_{Th232\,max} = \max_{r \in Th232-chain}\{UL_r\} \qquad (3)$$

Similarly, the exceedance of a supervision limit C can be established by comparison with the sum of the values

$$C^L_{U238\,max} = \max_{r \in U238-chain}\{LL_r\}, \quad C^L_{Th232\,max} = \max_{r \in Th232-chain}\{LL_r\} \qquad (4)$$

Including into the considerations also the test values $C^P_{U238max}$ and $C^P_{Th232max}$, which are defined similarly to Eqs. (3) and (4) by means of the most probable expectation values of specific activity PL_r, the following three situations may arise as schematically shown in Fig. 2.

In *case 1*, the compliance with the supervision limit is shown sufficiently reliable. The investigated waste batch does not require radiological supervision.

In *case 3*, the sum of the maximum values $C_{U238max}$ and $C_{Th232max}$ of specific activities of the long-lived radionuclides of the U-238 and Th-232 decay chains exceeds with high confidence the supervision limit. Thus, the investigated batch requires radiological supervision.

In *case 2*, the condition defined in Eq. (1) is not observed, but it is still not possible to reach a reliable conclusion about the need for radiological supervision. If justifiable from an economic point of view, the sample number should be in-

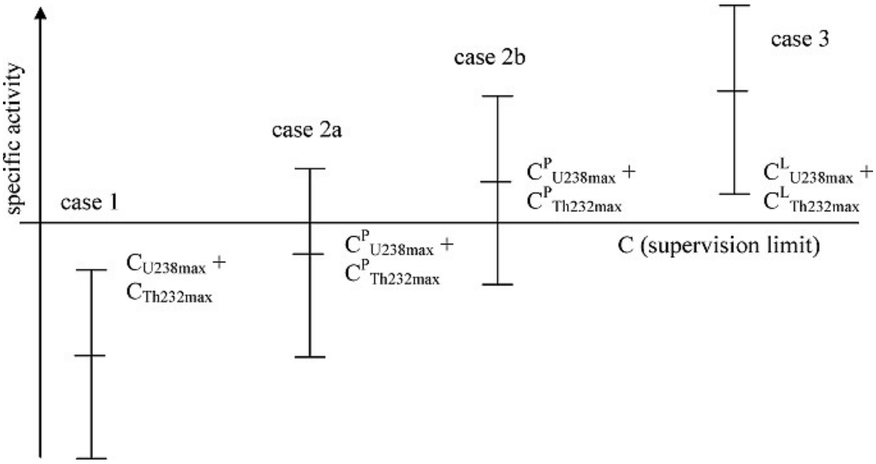

Fig. 2. Schema of possible situations concerning the proof of compliance with a supervision limit C based on interval estimators for the expectation value of specific activity

creased (from n to n′ > n) to get a decisive result about the compliance or exceedance of the supervision limit. The width of the confidence interval increases with the variance of the sample measurements, but it decreases with the sample number. If one assumes that the estimated values of the statistical distribution parameters will not change substantially after an increase of the sample number, the status of the most probable expectation values "$C^P_{U238max} + C^P_{Th232max}$" compared to the supervision limit will remain unchanged, while the width of the confidence interval will be reduced. Thus, in the case 2a it is possible to forecast a sample number n′, which should be sufficient for reducing the statistical uncertainty to a level that enables a reliable prove of compliance with the supervision limit. Similarly, in case 2b, the sample number can be increased such that the exceedance of the supervision limit can be shown with high probability. Of course, the increase of the sample number may result in changes of the formerly estimated values of the statistical parameters, leading to a shift of "$C^P_{U238max} + C^P_{Th232max}$" compared to the supervision limit. But, also in this case the width of the confidence interval is likely to be reduced by increasing the sampling number. The resulting estimates of confidence limits produce a new picture similar to that of Fig. 2. For adverse circumstances a renewed increase of the sample number could be necessary to enable a reliable radiological assessment of waste batch.

4 Calculation of distribution parameters and confidence limits

The calculation of confidence limits for the expectation value of a random variable on the basis of a certain sample requires the knowledge (or a plausible assumption) about the type of the statistical distribution of the investigated population (here: distribution of the specific activity C_r of the radionuclide indexed by r, in the considered batch).

In general it can be assumed that the specific activity C_r follows a lognormal distribution, which may be disturbed by a constant background value. This assumption is expressed by $C_r \sim c_r + \mathrm{Ln}(\mu_r, \sigma_r)$, where c_r denotes the background value. This is a non-negative quantity, which is bounded from above by the smallest measurement: $0 \leq c_r < C_{r,<1>}$. The parameters μ_r and σ_r denote the expectation value and the standard deviation of the quantity $X_r = \ln(C_r - c_r)$, which satisfies a normal distribution; $X_r \sim N(\mu_r, \sigma_r)$. A constant background value c_r of the specific activity may be due to the geogenic origin of the material or results from technological processes or mixing of samples, respectively. (Note: By a perfect mixing of the waste material the specific activity will approach a constant value, $C_r = c_r$).

Under certain circumstances, the specific activity of waste material can be approximated by means of a normal distribution. This regards relatively homogeneous wastes and also may apply to mixed samples produced by a large number of spot samples. Such cases are described by $C_r \sim N(\mu_r, \sigma_r)$.

To determine the most adequate type of statistical distribution, one can compare the coefficients of determination $R^2_{(N)}$ and $R^2_{(Ln)}$ of quantile-quantile-plots of the ordered measurements $C_{r;<i>}$ (i = 1 to n) and their logarithms $\ln(C_{r;<i>})$ over the respective quantiles k_{Pi} of the standard normal distribution. For the example of U-238 and Ra-226 measurements given in Tab. 1 for a sample number of n = 20, this comparison is shown in Fig. 3 for Ra-226. In the considered case, the specific activity has to be approximated by a lognormal distribution.

Table 1. Ordered measurements $C_{r,<i>}$ (i = 1 to n) of the specific activity of U-238 and Ra-226 (in Bq/g); quantiles k_{Pi} of the standard normal distribution; n = 20; Pi = (2 · i – 1)/(2 · n)

<i>	k_{Pi}	$C_{U238,<i>}$	$C_{Ra226,<i>}$	<i>	k_{Pi}	$C_{U238,<i>}$	$C_{Ra226,<i>}$
1	–1.960	0.12	0.14	11	0.063	0.38	0.47
2	–1.440	0.16	0.23	12	0.189	0.41	0.54
3	–1.150	0.19	0.25	13	0.319	0.51	0.67
4	–0.935	0.22	0.26	14	0.454	0.52	0.69
5	–0.755	0.24	0.27	15	0.598	0.69	0.93
6	–0.598	0.30	0.34	16	0.755	0.91	1.06
7	–0.454	0.30	0.37	17	0.935	1.06	1.27
8	–0.319	0.31	0.40	18	1.150	1.12	1.34
9	–0.189	0.33	0.41	19	1.440	1.76	2.03
10	–0.063	0.33	0.42	20	1.960	2.49	3.23

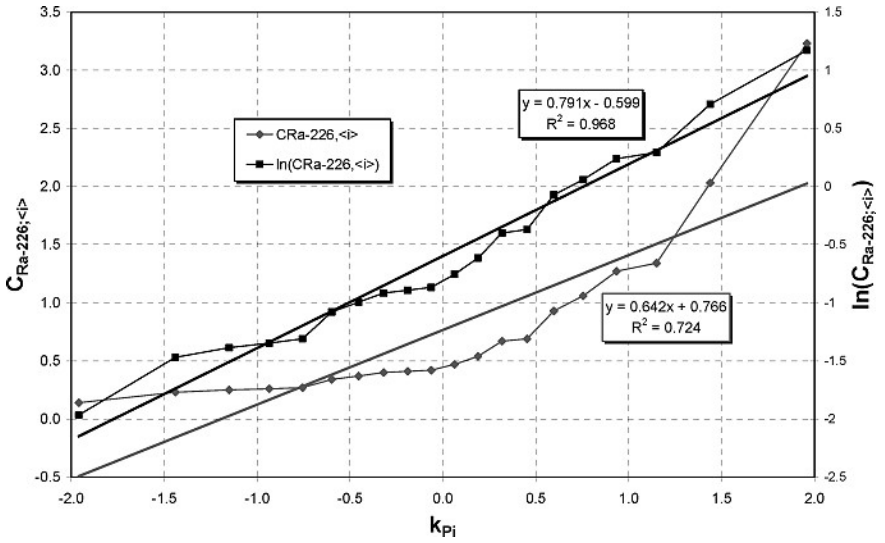

Fig. 3. Q-Q-Plot of Ra-226 measurements given in Tab. 1 (in Bq/g) and of their logarithms

The methods for calculating the statistical parameters of normally distributed measurements $C_r \sim N(\mu_r, \sigma_r)$ and for the 3-parametric lognormal distribution $C_r \sim c_r + Ln(\mu_r, \sigma_r)$ are described and illustrated by means of examples in detail in [2]. Measurements with the result "specific activity $C_{r,i}$ is less than the decision threshold $C^*_{r,i}$" can be taken into account on the basis of a method described in [3]. Outliers are recommended to be identified by the Grubbs test [4]. The recommendation [2] also comprises detailed explanations and calculations of the upper and lower confidence limits UL_r and LL_r and of the most probable values PL_r of the average specific activity.

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Experience from the Third International Nuclear Emergency Exercise (INEX 3) on Consequence Management.
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Since the beginning of the 1990s, the OECD Nuclear Energy Agency (NEA) has offered its member countries a forum for improving efficiency and effectiveness in nuclear emergency management, focusing in particular on the international aspects of emergency preparedness and response. A central approach to this has been the preparation and conduct of the International Nuclear Emergency Exercise (INEX) series.

The INEX 3 consequence management exercise series was developed by the NEA Working Party on Nuclear Emergency Matters in response to its members' desire to better prepare for the longer-term response following a nuclear or radiological emergency. INEX 3 was designed and conducted to allow participants to investigate the national and international arrangements for responding to widespread radiological contamination of the environment and the consequence management issues likely to be raised in the medium to long term following such an event. The main areas addressed included agriculture and food countermeasures, decisions on countermeasures such as travel, trade or tourism, recovery management and public information. The exercise series, conducted in 2005–2006, was followed by an evaluation workshop aimed at allowing participants to share their national experiences with INEX 3, compare approaches, analyse the implications on decision making and identify key needs in longer-term consequence management.

This report summarises the development of the INEX 3 exercise, the major evaluation outcomes of the national exercises, and the key policy-level outcomes, recommendations and follow-up activities arising from the exercise and workshop.

Based on the review of each identified issue to determine its current status internationally, to identify gaps and areas for improvement, and investigate any linkages into other related projects, a series of INEX 3 follow-up activities was identified that would bring value to the international community, including:

- *Expert Group on Recovery, Agriculture, and Food Countermeasures* to develop a report providing experience exchange on national and international strategies, approaches and considerations adopted; identified gaps; and other relevant issues in recovery, agricultural and food countermeasures. This work will be carried out through information exchange methods, and supported by an analysis and correlation of existing case studies in recovery and agricultural management (radiological and other) with the INEX 3 outcomes to extract key lessons for future consequence and recovery management.
- *Expert Group on Soft Countermeasures* to develop strategies and approaches for possible actions of relevance and common considerations, and undertake information exchange to move towards compatible decision making on these types of countermeasures.

The outcomes of these activities will be shared broadly with the international community.